hour. These are "negative" currents, *i.e.* they flow from the cold to the hot electrode across the gap. Their magnitude is somewhat greater in hydrogen than in nitrogen.

(4) These large negative currents appear to be intimately associated with the transit across the electrode gap, first of the impurities in the carbon, and afterwards of the carbon itself.

(5) Small "positive" currents of a few micro-ampères have been detected with new (but not with old) carbon electrodes at the lower stages of the heating.

(6) Some of these effects have also been obtained from non-electric sources of heat.

The Wave-Problem of Cauchy and Poisson for Finite Depth and slightly Compressible Fluid.

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1. *Introduction.*

The present paper is in some respects a completion of a former paper* on water waves resulting from a given disturbance. The following article is devoted to a numerical discussion of a solution, previously given, of the normal Cauchy-Poisson problem for finite constant depth of fluid. The last part of the paper contains a detailed treatment of compressible fluids, with a view to elucidating the initial stages of the spreading out of a disturbance initially confined to a limited region of the fluid. It is found that a very general case of propagation is capable of formal solution.

2. *Numerical Discussion of the Cauchy-Poisson Problem for Finite Depth.*

The serial solution given in the previous paper lends itself to a certain extent to numerical treatment, though not so well as for the case of infinite depth, which has been so completely discussed by Lamb.† In the general case there does not seem to be any general transformation to facilitate the calculation, so that we have to rely on the direct use of the series. The solution referred to may be briefly recapitulated as follows:—

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* 'Roy. Soc. Proc.,' A, 1910, vol. 83, p. 347.

† H. Lamb, 'Proc. Lond. Math. Soc.,' Series 2, 1904, vol. 2, p. 371.

Let the motion be in two dimensions x, y, and let h be the constant depth of the fluid. It is required to find the elevation η at time t at a point distant x from the origin on the free surface $y = 0$, when the motion is caused by the release from rest at time $t = 0$ of a mass of fluid of cross-section Q heaped up on a very small base near the origin. Writing ϕ_λ for $\cot^{-1}(2\lambda h/x)$, and $\{n, \lambda\}$ for the coefficient of x^λ in the expansion of $(1+x)^n(1-x)^{-n}$, we have

$$\frac{\pi x\eta}{Q} = -\Omega_1\left(\frac{gt^2}{2x}\right)+\frac{\Omega_2}{1.3}\left(\frac{gt^2}{2x}\right)^2-\ldots+(-)^n\frac{\Omega_n}{1.3\ldots(2n-1)}\left(\frac{gt^2}{2x}\right)^n+\ldots,$$

where

$$\Omega_n = \sum_{\lambda=1}^{\infty}(-)^\lambda\{n, \lambda\}\cos(n+1)\phi_\lambda\sin^{n+1}\phi_\lambda+\left[\begin{array}{l}0, \text{ if } n \text{ is even,}\\ (-1)^{\frac{1}{2}(n+1)}, \text{ if } n \text{ is odd.}\end{array}\right.$$

The calculations were performed for the three values $\frac{1}{2}$, 1, and ∞ of h/x, the latter serving as a partial check on the accuracy of the work. The series for Ω_n converges very slowly if h/x is taken much less than $\frac{1}{2}$, while the final series for η converges slowly when t is large. It was found possible to take values of $t(g/2x)^{\frac{1}{2}}$ up to 4, but to go much further would necessitate not only the taking of more terms of each series, but also the use of more figures in the logarithms at every stage. The calculation of $\{n, \lambda\}$ was carried out by repeated application of the formula

$$\{n+1, \lambda\} = \{n+1, \lambda-1\}+\{n, \lambda\}+\{n, \lambda-1\}.$$

The values used corresponded roughly to $n+\lambda \leq 25$, the value of $\{10, 10\}$ being checked independently.

The following table gives the numbers as they actually came out in calculation :—

$\pi x\eta/Q$.

$t(g/2x)^{\frac{1}{2}}$.	$h = \infty$.	$h = x$.	$h = \frac{1}{2}x$.	$t(g/2x)^{\frac{1}{2}}$.	$h = \infty$.	$h = x$.	$h = \frac{1}{2}x$.
1	0·93	1·13	0·79	2·75	−3·54	−1·58	−18·88
1·25	1·31	1·59	0·99	3	−3·23	−0·05	−26·26
1·5	1·55	1·96	0·84	3·25	−0·99	5·92	
1·75	1·42	1·97	− 0·19	3·5	2·52	11·44	
2	0·70	1·47	− 2·45	3·75	3·48	17·04	
2·25	−0·64	0·26	− 6·40	4	3·4	29	
2·5	−3·31	−1·06	−12·03				

The comparison with Lamb's results shows that the numbers in the first column are probably accurate to 2 per cent., but that for $t(g/2x)^{\frac{1}{2}} = 4$ serious errors are introduced (by the use of four-figure logarithms). The figure below shows the rise of the surface when $x = \frac{1}{2}g$ for values of t from 0 to 3·5, taking $Q = \pi x/10$.

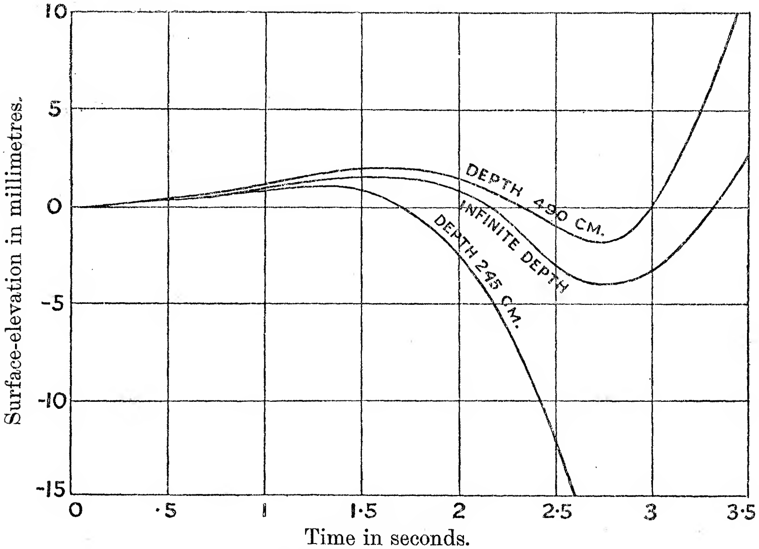

Fig. 1.—Surface-elevation at a fixed position at various times. $x = 490$ cm., initial area of fluid heaped up 154 sq. cm.

For depth 490 cm. a tidal wave of velocity $(gh)^{\frac{1}{2}}$ would arrive from the origin in 0·707 seconds, and for depth 245 cm. in one second. The peculiarities of very shallow water propagation have evidently not begun to appear for $h = \frac{1}{2}x$: when h is very small we should have nearly non-dispersive propagation with velocity $(gh)^{\frac{1}{2}}$. The surface-elevation at a given time is profoundly altered by making the depth finite, and seems to tend to lose its periodic character. It is remarkable that when $h = x$ the fluid actually begins to rise more quickly than when $h = \infty$. This is easily explained by means of Lord Rayleigh's formula* for the first term in η in ascending powers of t. Regarding h as the only variable, the coefficient of t^2 is proportional to $\theta^2 \cosh \theta / \sinh^2 \theta$, where $\theta = \pi x/2h$. Its progress as a function of h is shown in fig. 2.

3. *Wave Motion in Heavy Compressible Fluid.*

There are two main cases of wave motion in fluid, namely, when the fluid starts to move from rest in a given configuration, and when it is set in motion by given impulses. The former problem presents features of some difficulty. In discussing it it is usual to assume that the fluid is incompressible, in which case the solution is known. It is, however, necessary to postulate a certain initial distribution of pressure throughout the fluid, which may be explained by saying that pressure is transmitted instantaneously in an incompressible fluid. No insight into the process of transmission is gained in this way, as the course of the infinitely quick redistribution of pressure cannot be followed mathematically. The assumption of absolute incompressibility also makes any specification of the initial configuration indeterminate; for it is evident that a homogeneous incompressible fluid only appears to be disturbed when its free surface is ruffled, so that all initial disturbances have to be described in terms of the elevation of the free surface.

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* Lord Rayleigh, 'Phil. Mag.' [6], 1909, vol. 18, p. 4.

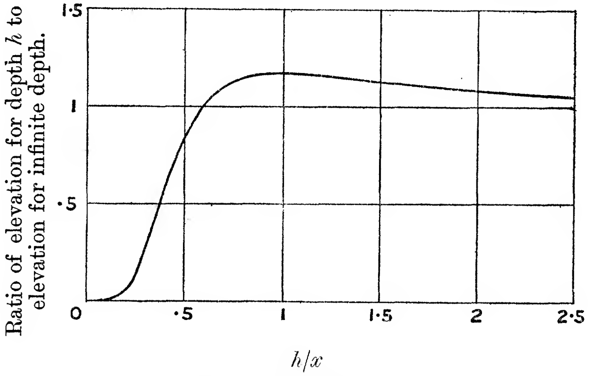

FIG. 2.—Elevation at a fixed distance x from the origin, and for a fixed small time t, expressed as a function of the depth h of the fluid.

The most natural way out of these difficulties seemed to the writer to be to consider, in the first place, a compressible fluid, and afterwards to pass to the limit when the compressibility is zero. A sufficient account of the matter will be given by the assumption p = constant + $a^2\rho$ of the previous paper; an incompressible fluid appears in the limit when a is infinite.

The essential features are illustrated well enough in two dimensions, which is the only case that will be considered. It has been found possible to obtain a result corresponding to very general initial conditions; in specialising the formulæ no attention has been paid to restrictions of a functional kind necessary to secure the validity of the theorems used.

For a finite constant depth h of fluid the immediate conditions of the problem are :—

(i) The differential equation

$$\frac{\partial^2\phi}{\partial t^2} = a^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+2\gamma\frac{\partial\phi}{\partial y}\right), \quad \text{where} \quad \gamma = g/2a^2; \tag{1}$$

(ii) The condition at the free surface,

$$\partial^2\phi/\partial t^2 = g\,\partial\phi/\partial y \quad \text{when} \quad y=0;$$

(iii) The condition

$$\partial\phi/\partial y = 0 \quad \text{when} \quad y = h;$$

(iv) The condition of initial rest,

$$\phi = 0 \quad \text{when} \quad t = 0.$$

The surface elevation at time t is $\eta = g^{-1}(\partial\phi/\partial t)_{y=0}$. In the present case we have to specify not only η, but also the *state of every element of fluid* at time $t = 0$. Since $\int\rho^{-1}dp = \partial\phi/\partial t + gy$, this is done by fixing the initial value of $\partial\phi/\partial t$ at every point; the initial value of η then follows as a necessary consequence. Let us therefore take as the last condition

$$\text{(v)} \qquad \frac{e^{\gamma y}}{g}\left(\frac{\partial\phi}{\partial t}\right)_{t=0} = \theta(x, y),$$

where $\theta(x, y)$ is an arbitrary function of x and y.

The suitable particular solutions of (i) to (iv) have already been given.* Their general type is given by

$$\phi e^{\gamma y} = (\mathrm{A}e^{\mu y} + \mathrm{B}e^{-\mu y})\frac{\sin\sigma t}{\sigma}\cos kx,$$

where $$\sigma^2/a^2 = k^2 - \mu^2 + \gamma^2,$$

or by $$\phi e^{\gamma y} = (\mathrm{A}\cos\nu y + \mathrm{B}\sin\nu y)\frac{\sin\sigma t}{\sigma}\cos kx,$$

where $$\sigma^2/a^2 = k^2 + \nu^2 + \gamma^2.$$

We may refer to these briefly as the μ and ν solutions respectively. The admissible values of μ and ν are given by the equations

$$\mu\coth\mu h = \gamma(k^2 + \mu^2 - \gamma^2)/(k^2 - \mu^2 + \gamma^2)$$

and $$\nu\cot\nu h = \gamma(k^2 - \nu^2 - \gamma^2)/(k^2 + \nu^2 + \gamma^2).$$

respectively. The value of ν may, of course, be regarded as a purely imaginary value of μ. It does not seem easy to show directly that the above equation for μ has no complex roots of the form $a + ib$, but this can be shown by general considerations. For, if we assimilate the fluid to a dynamical system, and consider the modes of vibration involving x in the connexion $\cos kx$, the corresponding values of σ are purely real, or at most purely imaginary; and, since $\sigma^2/a^2 = k^2 - \mu^2 + \gamma^2$, the same is true of μ.

It is remarkable that Lord Kelvin's particular solutions for deep incompressible fluid,† as well as the normal Cauchy-Poisson solution, apply equally to the present ideal fluid. They are not, however, general enough for our present purpose, which is to satisfy the fifth condition of the problem as well as the first four.

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* P. 355 of the previous paper.

† Lord Kelvin, 'Phil. Mag.' [6], 1907, vol. 13, p. 1; 'Collected Papers,' vol. 4.

4. *Compressible Fluid, Infinite Depth.*

When h tends to infinity there is always the root $\mu = \gamma$, and also the root $\mu = k - \gamma$ if $k > \gamma$. The former gives $\phi = \cos kx \sin kat$, and corresponds to sound-waves propagated horizontally, but it will not be required here. The root $\mu = k - \gamma$ gives $\sigma^2 = gk$. The values of ν become heaped more and more together, and in the limit ν can take any value. The fundamental ν-solution can be written in the form

$$\frac{\phi e^{\gamma y}}{g} = \{2\nu\gamma \cos \nu y - (k^2+\nu^2-\gamma^2)\sin \nu y\} \frac{\sin \sigma t}{\sigma} \cos k(x-\xi),$$

where

$$\sigma^2 = a^2(k^2+\nu^2+\gamma^2). \tag{2}$$

It is to be expected that on generalising these solutions to fit the condition (v) we should obtain a solution in the form of a ν-integral, with an extra term corresponding to the μ-root when $k > \gamma$. In fact, the solution satisfying the condition (v) is

$$\frac{\pi\phi e^{\gamma y}}{g} = \frac{2}{\pi}\int_0^\infty d\nu \int_0^\infty A \frac{\sin \sigma t}{\sigma} \frac{\{2\nu\gamma \cos \nu y - (k^2+\nu^2-\gamma^2)\sin \nu y\}}{(k^2+\nu^2-\gamma^2)^2+4\nu^2\gamma^2}\, dk$$
$$+ 2\gamma \int_\gamma^\infty B \frac{k-\gamma}{k} \frac{\sin (gk)^{\frac{1}{2}} t}{(gk)^{\frac{1}{2}}} e^{-(k-\gamma)y}\, dk, \tag{3}$$

where*

$$\left.\begin{aligned} A &= \int_{-\infty}^\infty \int_0^\infty \cos k(x-\xi)\,\{2\nu\gamma\cos\nu\eta - (k^2+\nu^2-\gamma^2)\sin\nu\eta\}\,\theta(\xi,\eta)\,d\xi\,d\eta, \\ B &= \int_{-\infty}^\infty \int_0^\infty \cos k(x-\xi)\, e^{-(k-\gamma)\eta}\,\theta(\xi,\eta)\,d\xi\,d\eta. \end{aligned}\right\} \tag{4}$$

In order to verify this we have to prove that

$$\pi\theta(x,y) = \frac{2}{\pi}\int_0^\infty d\nu \int_0^\infty A \frac{2\nu\gamma\cos\nu y - (k^2+\nu^2-\gamma^2)\sin\nu y}{(k^2+\nu^2-\gamma^2)^2+4\nu^2\gamma^2}\, dk$$
$$+ 2\gamma\int_\gamma^\infty B\frac{k-\gamma}{k} e^{-(k-\gamma)y}\, dk.$$

The proof depends on a theorem of Orr,† who has given a very valuable extension of Fourier's theorem of the type required in this problem. Recalling Fourier's integral theorem in the form

$$\pi\theta(x, y) = \int_0^\infty dk \int_{-\infty}^\infty \cos k(x-\xi)\,\phi(\xi, y)\, d\xi,$$

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* The symbol η is here used in a new sense.

† W. M'F. Orr, "Extensions of Fourier's and the Bessel-Fourier Theorems," 'Royal Irish Acad. Proc.,' A, 1909, vol. 27, pp. 207—209.

we see that we have to prove that

$$\theta(\xi, y) = \frac{2}{\pi}\int_0^\infty \frac{2\,\nu\gamma\cos\nu y-(k^2+\nu^2-\gamma^2)\sin\nu y}{(k^2+\nu^2-\gamma^2)^2+4\nu^2\gamma^2}\,d\nu$$

$$\int_0^\infty\{2\nu\gamma\cos\nu\eta-(k^2+\nu^2-\gamma^2)\sin\nu\eta\}\,\theta(\xi,\eta)\,d\eta \qquad (5)$$

for $k<\gamma$, and that for $k<\gamma$ the term

$$2\gamma\frac{k-\gamma}{k}e^{-(k-\gamma)y}\int_0^\infty e^{-(k-\gamma)\eta}\theta(\xi,\eta)\,d\eta \qquad (6)$$

must be added on the right-hand side.

The result (5) is a case of Orr's theorem, allowing for the difference in notation; the condition that the roots of $C(\lambda)+iS(\lambda)=0$ should have negative imaginary parts reducing to the condition $k<\gamma$. The root $\lambda = i(k-\gamma)$, which in our notation is $\mu = i\nu = -(k-\gamma)$, gives rise, when $k>\gamma$, to a pole inside the contour used in Orr's proof. The residue of this pole is easily shown to cause the addition (6) to the right-hand side of (5).

Now let us confine the initial disturbance to the neighbourhood of the point $(0, u)$ within the fluid. Thus $\theta(x, y)$ is sensible only near this point, and we shall make it of such a magnitude that the double integral $\iint\theta(x,y)\,dx\,dy$ has the finite value $1/2\gamma = a^2/g$. The solution becomes

$$\frac{\pi\phi e^{\gamma y}}{g} =$$

$$\frac{1}{\pi\gamma}\int_0^\infty d\nu\int_0^\infty\frac{\{2\nu\gamma\cos\nu u-(k^2+\nu^2-\gamma^2)\sin\nu u\}\,\{2\nu\gamma\cos\nu y-(k^2+\nu^2-\gamma^2)\sin\nu y\}}{(k^2+\nu^2-\gamma^2)^2+4\nu^2\gamma^2}$$

$$\frac{\sin\sigma t}{\sigma}\cos kx\,dk+\int_\gamma^\infty\frac{k-\gamma}{k}e^{-(k-\gamma)(u+y)}\,\frac{\sin(gk)^{\frac{1}{2}}t}{(gk)^{\frac{1}{2}}}\cos kx\,dk. \qquad (7)$$

We can now pass to the limit when a is infinite, so that γ tends to zero. The first step is to prove that the first integral becomes indefinitely small. The most important term is

$$\frac{1}{\pi\gamma}\int_0^\infty\sin\nu u\sin\nu y\,d\nu\int_0^\infty\frac{\sin(k^2+\nu^2)^{\frac{1}{2}}at}{a(k^2+\nu^2)^{\frac{1}{2}}}\cos kx\,dk.$$

The integral with respect to k has the value*

$$\frac{\pi}{2a}J_0\{\nu(a^2t^2-x^2)^{\frac{1}{2}}\},$$

if $at>x$, and we may further replace it by its approximate value $\frac{\pi}{2a}J_0(\nu at)$;

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* Nielsen, 'Cylinderfunktionen,' p. 255.

or $\frac{1}{2}\left(\frac{\pi}{\nu a^3 t}\right)^{\frac{1}{2}}(\cos \nu a t + \sin \nu a t)$, since a is large. Hence the double integral becomes

$$\frac{1}{g}\left(\frac{a}{\pi t}\right)^{\frac{1}{2}} \int_0^{\infty} \frac{\sin \nu u \sin \nu y\,(\cos \nu a t + \sin \nu a t)}{\sqrt{\nu}}\, d\nu.$$

Now when λ is positive, $\int_0^{\infty} \begin{matrix}\cos\\ \sin\end{matrix} \lambda\nu \frac{d\nu}{\sqrt{\nu}} = \left(\frac{\pi}{2\lambda}\right)^{\frac{1}{2}}$, so that, expressing the products of trigonometrical functions as sums, we find that the above expression becomes

$$\frac{1}{4g}\left(\frac{2a}{t}\right)^{\frac{1}{2}} \{(at+y-u)^{-\frac{1}{2}} + (at-y+u)^{-\frac{1}{2}} - (at+y+u)^{-\frac{1}{2}} - (at-y-u)^{-\frac{1}{2}}\},$$

which tends to zero as a tends to infinity. The rest of the first integral is easily shown to vanish for $a = \infty$.

Hence, in the limit when the compressibility of the fluid is zero we have

$$\phi = \frac{g}{\pi}\int_0^{\infty} e^{-k(u+y)} \frac{\sin (gk)^{\frac{1}{2}} t}{(gk)^{\frac{1}{2}}} \cos kx\, dk, \tag{8}$$

which reduces to the integral solution of the Cauchy-Poisson problem when $u = 0$.

Returning to the equation (7), the first term may be described roughly as the spreading out of the initial pressure by sound-waves in the fluid, and the second as the Cauchy-Poisson term predominating after a time. It is interesting to verify that we are really dealing, for finite values of a, with a case of finite propagation, that is, to find the points of non-uniform convergence of the first integral in (7). As we are only concerned with large values of k and ν, we can just as well consider the integral

$$\int_0^{\infty} \sin \nu u \sin \nu y\, d\nu \int_0^{\infty} \frac{\sin \sigma t}{\sigma} \cos kx\, dk, \text{ where } \sigma^2 = a^2(k^2+\nu^2+\gamma^2).$$

The integral with respect to k is zero* when $at < x$, and when $at > x$ its value is $\frac{\pi}{2a} J_0\{(\nu^2+\gamma^2)^{\frac{1}{2}}(a^2t^2-x^2)^{\frac{1}{2}}\}$. Thus the convergence of the double integral is of the same kind as that of the single integral

$$\int^{\infty} \nu^{-\frac{1}{2}} \sin \nu u \sin \nu y\, \{\cos \nu (a^2t^2-x^2)^{\frac{1}{2}} + \sin \nu (a^2t^2-x^2)^{\frac{1}{2}}\}\, d\nu;$$

and this latter integral is non-uniformly convergent near the points where $(a^2t^2-x^2)^{\frac{1}{2}} = \pm y \pm u$, *i.e.* where $a^2t^2 = x^2 + (y \pm u)^2$. Hence one set of points of non-uniform convergence occurs where a sound-wave starting from $(0, u)$ with velocity a arrives after a time t has elapsed, as we should expect. The other set arises from the quasi-reflection of the disturbance at the free surface.

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* Nielsen, p. 252.

5. *Finite Depth, Small Compressibility.*

It is possible to give a formal solution of the problem when the depth of the fluid is finite, by using an extension of Fourier's series given in the latter part of Orr's paper. We shall find it convenient to express all the solutions in the exponential form by writing $\mu = i\nu$. The fundamental solution is

$$\frac{\phi e^{\gamma y}}{g} = \{\mathrm{P}e^{\mu y} + \mathrm{Q}e^{-\mu y}\} \frac{\sin \sigma t}{\sigma} \cos k(x-\xi),$$

where

$$\sigma^2 = a^2(k^2-\mu^2+\gamma^2), \tag{9}$$

and the conditions satisfied by P and Q are

$$\mathrm{P}\{k^2-(\mu-\gamma)^2\} + \mathrm{Q}\{k^2-(\mu+\gamma)^2\} = 0$$

and

$$\mathrm{P}(\mu-\gamma)e^{\mu h} - \mathrm{Q}(\mu+\gamma)e^{-\mu h} = 0.$$

Writing

$$\left.\begin{aligned} \alpha(\mu, y) &\equiv e^{-\mu(h-y)}(-\mu-\gamma) - e^{\mu(h-y)}(\mu-\gamma) \\ \beta(\mu, u) &\equiv e^{\mu u}\{k^2-(\mu+\gamma)^2\} - e^{-\mu u}\{k^2-(\mu-\gamma)^2\} \\ \zeta(\mu) &\equiv e^{\mu h}(\mu-\gamma)\{k^2-(\mu+\gamma)^2\} + e^{-\mu h}(\mu+\gamma)\{k^2-(\mu-\gamma)^2\} \end{aligned}\right\}, \tag{10}$$

the solution for finite depth is

$$-\frac{\pi\phi e^{\gamma y}}{g} = \Sigma\alpha(\mu, y)\int_0^\infty \frac{1}{\zeta'(\mu)}\frac{\sin \sigma t}{\sigma}\,dk \int_{-\infty}^{\infty} \cos k(x-\xi)\,d\xi \int_0^h \beta(\mu, u)\,\theta(\xi, u)\,du. \tag{11}$$

The summation is extended over the roots of $\zeta(\mu) = 0$, which are precisely the quantities μ and ν already employed. The process of verifying this solution is the same as before. Using Fourier's integral theorem, we find that we have to prove that

$$-\theta(\xi, y) = \Sigma\frac{\alpha(\mu, y)}{\zeta'(\mu)}\int_0^h \beta(\mu, u)\,\theta(\xi, u)\,du$$

for $0 < y < h$. This is included in Orr's result.*

The case which is easiest to interpret is that of a concentrated disturbance in nearly incompressible fluid. When γ is small it is superfluous to attend to the restriction that $k > \gamma$, and it is easy to see that there is one μ-root nearly equal to k. Its approximate value is $k - \gamma\tanh kh$, giving $\sigma^2 = gk\tanh kh$ nearly. As before, the initial disturbance will be located near $(0, u)$ and such that $\iint \theta(x, y)\,dx\,dy$ is equal to a^2/g, in this case a large quantity. The part of ϕ corresponding to the above root is found to be

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* W. M'F. Orr, *loc. cit.*, p. 238. It must be borne in mind that only positive values of μ and ν are considered here, while Orr considers all the roots.

$$\frac{g}{\pi}\int_0^\infty \frac{\cosh k(h-u)\cosh k(h-y)}{\cosh^2 kh}\,\frac{\sin (gk\tanh kh)^{\frac{1}{2}}t}{(gk\tanh kh)^{\frac{1}{2}}}\cos kx\,dk. \tag{12}$$

The other roots tend to $\mu h = i(n+\frac{1}{2})\pi$, or $\nu h = (n+\frac{1}{2})\pi$, where $n = 0, 1, 2 \ldots$, the part of ϕ belonging to a particular one being

$$\frac{2a}{\pi h}\int_0^\infty \sin \nu u \sin \nu y \,\frac{\sin (k^2+\nu^2)^{\frac{1}{2}}\,at}{(k^2+\nu^2)^{\frac{1}{2}}}\cos kx\,dk,$$

that is

$$\frac{1}{h}\left(\frac{a}{\pi t}\right)^{\frac{1}{2}}\nu^{-\frac{1}{2}}\sin \nu u \sin \nu y\,(\cos \nu at + \sin \nu at). \tag{13}$$

The mere fact of making a large is not enough to make the aggregate of terms (13) negligible. In reality, they become large and quickly oscillatory for finite values of t. In a number of important cases, however, they are small and the velocity-potential is given by (12). They do not appear, for example, when either the source of disturbance or the point of observation is on the free surface of the fluid. The terms also become negligible when t is large, while the expression (12) increases without limit. This latter circumstance is, of course, due to the supposition that the initial elevation is confined to an infinitely small region near the origin, and would disappear on a more natural assumption.